Solutions of the focusing nonradial critical wave equation with the compactness property

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Abstract. Consider the focusing energy-critical wave equation in 3, 4 or 5 space dimensions. In a previous paper, we proved that any solution which is bounded in the energy space converges, along a sequence of times and in some weak sense, to a solution *with the compactness property*, that is a solution whose trajectory stays in a compact subset of the energy space up to space translation and scaling. It is conjectured that the only solutions with the compactness properties are stationary solutions and solitary waves that are Lorentz transforms of the formers. In this note we prove this conjecture with an additional non-degeneracy assumption related to the invariances of the elliptic equation satisfied by stationary solutions. The proof uses a standard monotonicity formula, modulation theory, and a new channel of energy argument which is independent of the space dimension.

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